FINALTERM EXAMINATION 2009

Calculus & Analytical Geometry-I

Time: 120 min
Marks: 80

Question No: 1 (Marks: 1) - Please choose one

If f is a twice differentiable function at a stationary point x_0 and $f''(x_0) > 0$ then f has relative ……. At x_0

- ► Minima
- ► Maxima
- ► None of these

Question No: 2 (Marks: 1) - Please choose one

In the notation

$$\int f(x)dx = F(x) + C$$

C represents

- ► A polynomial
- ► A Constant
- ► A Variable
- ► None of these

Question No: 3 (Marks: 1) - Please choose one

According to Power-Rule of differentiation, if $f(x) = x^n$ where n is a real number, then $\frac{d}{dx}[x^n] =$

- ► x^{n-1}
- ► $n\,x^{n-1}$
- ► $n\,x^{n+1}$

▶ $(n-1)x^{n+1}$

Question No: 4 (Marks: 1) - Please choose one

If $2x - y = -3$ then $\frac{dy}{dx} =$

▶ 2
▶ -2
▶ 0
▶ -3

Question No: 5 (Marks: 1) - Please choose one

30^0 = _____

▶ $\frac{\pi}{3}$
▶ $\frac{\pi}{4}$
▶ $\frac{\pi}{6}$
▶ $\frac{\pi}{2}$

Question No: 6 (Marks: 1) - Please choose one

If a function g is differentiable at a point x and a function f is differentiable at a point g(x), then the _____ is differentiable at point x .

▶ Composition (f o g)
▶ Quotient (f / g)
▶ Product (f . g)
▶ Sum (f + g)

Question No: 7 (Marks: 1) - Please choose one

Let a function f be defined on an interval, and let x_1 and x_2 denote points in that

interval. If $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ then which of the following statement is correct?

- ► f is an increasing function.
- ► f is a decreasing function.
- ► f is a constant function.

Question No: 8 (Marks: 1) - Please choose one

If $f''(x) < 0$ on an open interval (a,b) then which of the following statement is correct?

- ► f is concave up on (a, b).
- ► f is concave down on (a, b)
- ► f is linear on (a, b).

Question No: 9 (Marks: 1) - Please choose one

The sum $\sum_{k=1}^{n} f(x^*_k)\Delta x_k$ is known as:

- ► Riemann Sum
- ► General Sum
- ► Integral Sum
- ► Geometric Sum

Question No: 10 (Marks: 1) - Please choose one

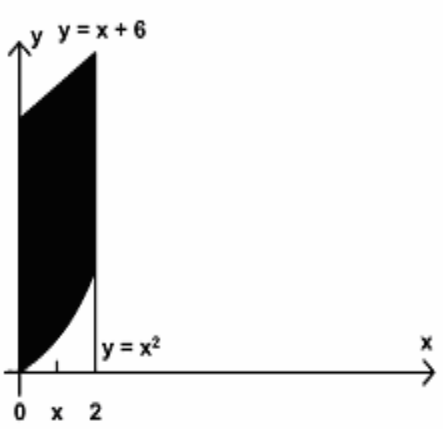
$$\sum_{k=1}^{n} f(x^*_k)\Delta x_k$$

What does 'n' represent in Riemann Sum ?

- ► No. of Circles
- ► No. of Rectangles
- ► No. of Loops
- ► No. of Squares

Question No: 11 (Marks: 1) - Please choose one

What is the area of the region in the following figure?



$A=\int_{0}^{2}\left[(x+6)-\left(x^{2}\right)\right]dx$

▶

$A=\int_{x}^{2}\left[(x+6)-\left(x^{2}\right)\right]dx$

▶

$A=\int_{0}^{2}\left[(x+6)+\left(x^{2}\right)\right]dx$

▶

$A=\int_{0}^{x}\left[(x+6)-\left(x^{2}\right)\right]dx$

▶

Question No: 12 (Marks: 1) - Please choose one

If $\int_{1}^{4} f(x)\,dx = 2$ and $\int_{1}^{4} g(x)\,dx = 10$ then which of the following is value of $\int_{1}^{4} [3f(x) - g(x)]\,dx$?

▶ 16
▶ 12
▶ -4
▶ -8

Question No: 13 (Marks: 1) - Please choose one

$\int_{0}^{1} 2x(x^{2}+4)dx =$ _____

$\frac{9}{2}$

▶

$\frac{5}{2}$

▶

$\frac{2}{5}$

▶

$\frac{-9}{2}$

▶

Question No: 14 (Marks: 1) - Please choose one

Let f is a smooth function on [0, 3]. What will be the arc length L of the curve y = f(x) from
x = 0 to x = 3?

$$L = \int_0^3 \sqrt{1+[f(x)]^2}\,dy$$

▶

$$L = \int_a^b \sqrt{1+[f'(x)]^2}$$

▶

$$L = \int_0^3 \sqrt{1+[f'(x)]^2}\,dy$$

▶

$$L = \int_0^3 \sqrt{1+[f'(x)]^2}\,dx$$

▶

Question No: 15 (Marks: 1) - Please choose one

Let f be a smooth, nonnegative function on [1, 3]. What is the surface area S generated by revolving the portion of the curve y = f(x) between x = 1 and x = 3 about the x-axis?

$$S = \int_0^2 2\sqrt{1+[f(x)]}dx$$

▶

$$S = \int_0^3 2\pi f(x)\sqrt{1+[f'(x)]}dx$$

▶

$$S = \int_0^2 2\sqrt{1+[f'(x)]}dx$$

▶

$$S = \int_1^3 2\pi f(x)\sqrt{1+[f'(x)]^2}dx$$

▶

Question No: 16 (Marks: 1) - Please choose one

Let an object is displaced 2m by a force of 2N. What is the work done W?

▶ - 4
▶ 4
▶ 2
▶ 0

Question No: 17 (Marks: 1) - Please choose one

$$\int_a^{+\infty} f(x)dx = \lim_{l\to\infty}\int_\alpha^l f(x)dx$$

Consider the improper integral if the limit exists then which of the following can be occured?

▶ Diverges
▶ Converges
▶ Test fail

Question No: 18 (Marks: 1) - Please choose one

If f is continuous on (a, b] but does not have a limit from the right then the integral

$$\int_a^b f(x)dx = \lim_{l\to a}\int_l^b f(x)dx$$

defined by is called **:**

▶ Improper

► Proper
► Line

Question No: 19 (Marks: 1) - Please choose one

For a sequence $\{a_n\}$ if the difference between successive terms $a_{n+1} - a_n < 0$ then the sequence is known as **:**

► Increasing
► Decreasing
► Nondecreasing
► Nonincreasing

Question No: 20 (Marks: 1) - Please choose one

For a sequence $\{a_n\}$ if the ratio of successive terms $\frac{a_{n+1}}{a_n} > 1$ then the sequence is known as**:**

► Increasing
► Decreasing
► Nondecreasing
► Nonincreasing

Question No: 21 (Marks: 1) - Please choose one

Which of the following is true for the sequence $\{n\}_{n=0}^{\infty}$?

► Nonincreasing
► Nondecreasing
► Increasing
► Decreasing

Question No: 22 (Marks: 1) - Please choose one

If $f(n) = a_n$ is the nth term of the sequence and f is differentiable and $f'(n) \leq 0$ then the sequence will be :

► Increasing
► Decreasing
► Nondecreasing

► Nonincreasing

Question No: 23 (Marks: 1) - Please choose one

If Newton's Method is used to approximate the real solutions of the equation $x^3 + x - 3 = 0$ and the first guess $x_1 = 1$, What is x_2 ?

► $\frac{5}{4}$

► $\frac{1}{4}$

► $\frac{-1}{2}$

► $\frac{3}{4}$

► $\frac{3}{2}$

Question No: 24 (Marks: 1) - Please choose one

Suppose that we apply Newton's Method to approximate the real solutions of the equation $x^3 - 2x^2 - 1 = 0$. If we start at $x_1 = 2$, then which of the following is value of x_2 ?

► 6

► 2.25

► 0

► 2

Question No: 25 (Marks: 1) - Please choose one

If the sequence of partial sum of a series converges then what will the series show itself ?

► Diverges

► Converges

► Gives no information

Question No: 26 (Marks: 1) - Please choose one

The series $\sum u_k$ be a series with positive terms and suppose that $\rho = \lim_{k\to\infty} \frac{u_{k+1}}{u_k}$ if $\rho > 1$, then which of the following is true?

- ▶ Converges
- ▶ Diverges
- ▶ May converges or diverges
- ▶ Gives no information

Question No: 27 (Marks: 1) - Please choose one

The series $\sum u_k$ be a series with positive terms and suppose that $\rho = \lim_{k\to\infty} \frac{u_{k+1}}{u_k}$ if $\rho = 1$, then which of the following is true?

- ▶ Converges
- ▶ Diverges
- ▶ May converges or diverges
- ▶ Gives no information

Question No: 28 (Marks: 1) - Please choose one

The series $\sum u_k$ be a series with positive terms and suppose that $\rho = \lim_{k\to\infty} \sqrt[k]{u_k} = \lim_{k\to\infty} (u_k)^{\frac{1}{k}}$ if $\rho = 1$, then which of the following is true?

- ▶ Converges
- ▶ Diverges
- ▶ May converges or diverges
- ▶ Gives no information

Question No: 29 (Marks: 1) - Please choose one

For an alternating series to be convergent which of the following condition must be satisfied?

- ▶ $\lim_{k\to\infty} a_k = 1$
- ▶ $a_1 > a_2 > a_3 ..... > a_k > ....$
- ▶ $a_1 \le a_2 \le a_3 ..... \le a_k \le ....$
- ▶ Gives no information

Question No: 30 (Marks: 1) - Please choose one

For an alternating series to be convergent which of the following condition must be satisfied?

- ▶ $a_1 \geq a_2 \geq a_3 ..... \geq a_k \geq ....$
- ▶ $\lim_{k \to \infty} a_k = 0$
- ▶ $a_1 \leq a_2 \leq a_3 ..... \leq a_k \leq ....$
- ▶ $\lim_{k \to \infty} a_k = 1$
- ▶

Question No: 31 (Marks: 1) - Please choose one

What is the base of natural logarithm?

- ▶ 2.71
- ▶ 10
- ▶ 5
- ▶ Any real number

Question No: 32 (Marks: 1) - Please choose one

A function F is called an antiderivative of a function f on a given interval if _____ $= f(x)$, for all x in that interval.

- ▶ $F'(x)$
- ▶ $F(x)$
- ▶ $f'(x)$
- ▶ $f''(x)$

Question No: 33 (Marks: 1) - Please choose one

$\log_b ac =$ _____

- $\log_b a + \log_b c$
- $\log_b a - \log_b c$
- $\frac{\log_b a}{\log_b c}$
- $(\log_b a)(\log_b c)$

Question No: 34 (Marks: 1) - Please choose one

$\log_b a^r =$ _____

- $a \log_b r$
- $r \log_b a$
- $\frac{\log_b a}{\log_b r}$
- $\log_b a + \log_b r$

Question No: 35 (Marks: 1) - Please choose one

$\log_b \frac{1}{c} =$ _____

- $\log_b c$
- $1 - \log_b c$
- $-\log_b c$
- $1 + \log_b c$

Question No: 36 (Marks: 1) - Please choose one

$\log_b \frac{1}{t} =$ _____

- $\log_b t$
- $1 - \log_b t$
- $1 + \log_b t$
- $-\log_b t$

Question No: 37 (Marks: 1) - Please choose one

What is the sum of following series?

$1^2 + 2^2 + 3^2 + 4^2 + \_\_\_\_\_ + n^2$

- $\frac{n(n+1)(2n+1)}{6}$
- $\frac{n(2n)(2n+1)}{6}$
- $\frac{(n+1)(n+2)}{2}$
- $\frac{(n+1)(2n+1)}{6}$

Question No: 38 (Marks: 1) - Please choose one

$\sum_{k=1}^{n} \frac{k^3}{2} =$ _____

$\frac{n(n+1)}{4}$

▶

$\frac{[n(n+1)]^2}{8}$

▶

$\frac{n(n+1)(2n+1)}{12}$

▶

Question No: 39 (Marks: 1) - Please choose one

Let $y = \frac{2\sqrt{2}}{3}x^{\frac{3}{2}} - 2x \;;\; 0 \le x \le 1$ then which of the following is the length of the curve?

$L = \int_0^1 \sqrt{1 + \left[\left(\frac{2\sqrt{2}}{3}x^{\frac{3}{2}} - 2x\right)\right]^2}\, dx$

▶

$L = \int_0^1 \sqrt{\left[\frac{d}{dx}\left(\frac{2\sqrt{2}}{3}x^{\frac{3}{2}} - 2x\right)\right]^2}\, dx$

▶

$L = \int \sqrt{1 + \left[\frac{d}{dx}\left(\frac{2\sqrt{2}}{3}x^{\frac{3}{2}} - 2x\right)\right]^2}\, dx$

▶

$L = \int_0^1 \sqrt{1 + \left[\frac{d}{dx}\left(\frac{2\sqrt{2}}{3}x^{\frac{3}{2}} - 2x\right)\right]^2}\, dx$

▶

Question No: 40 (Marks: 1) - Please choose one

If $f(x) = e^{-x}$ at $x = 0$ be the Taylor series, then which of the following is also true?

▶ Arithmetic series

▶ Maclaurin series

▶ Geometric series

▶ Harmonic series

Question No: 41 (Marks: 2)

$u = \frac{\pi}{2} - x$ $\int_0^{\pi} \sin(\frac{\pi}{2} - x)dx$

Using substitution transform the integral into variable u.

Question No: 42 (Marks: 2)

$\int_3^{+\infty} \frac{dx}{2x^2}$

Evaluate the improper integral

Question No: 43 (Marks: 2)

A function $f(x) = 6 - 2x - x^2$ has critical point 1 in an interval [-4, 3]. Find the absolute minimum value of the function.

Question No: 44 (Marks: 3)

Find the absolute maximum value of the function:

$f(x) = 2x^3 + 3x^2 - 12x + 4$ *on* $[-4, 2]$

Question No: 45 (Marks: 3)

Find the area of the region bounded by the curve $y = x^2 - 4x - 5$ and $y = x + 1$ (do not evaluate).

Question No: 46 (Marks: 3)

$\left\{\frac{3}{n^2}\right\}_{n=5}^{\infty}$

Determine whether the following sequence is strictly monotone:

Question No: 47 (Marks: 5)

Determine whether the sequence converges or diverges. If converges find limit

$\lim_{n\to\infty} \frac{3^n + (-1)^n}{3^{n+1} + (-1)^{n+1}}$

Question No: 48 (Marks: 5)

Find the lengths of the curves

$$x = \frac{t^2}{2}, \quad y = \frac{(2t+1)^{\frac{3}{2}}}{3} \ , \ 0 \le t \le 4$$

Question No: 49 (Marks: 5)

Evaluate the indefinite integral $\int \left[(x^4+2)\right]\left[\cos(x^5+10x)\right] dx$ by substitution method.

Question No: 50 (Marks: 10)

Find the Maclaurin series for $f(x) = e^{2x}$